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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%% Script to generate synthetic data with uniform distribution
%% to estimate the asymmetric relationship between the x and y
%% for positive and negative x's, and plot a scatter plot,
%%
%% Figure 1 of Frankignoul and Kwon (submitted to GRL)
%%
%% Frankignoul, C., and Y.-O. Kwon: On the statistical estimation of asymmetrical↙
relationship between two climate variables. Geophys. Res. Lett., submitted.
%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


clear

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Set variables
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

  p0=[];           %% slope for the symmetric regresssion
  p1p=[]; p1n=[];  %% slope for the method 1 asymmetric regression
  p2p=[]; p2n=[];  %% slope for the method 2 asymmetric regression
  c1p=[]; c1n=[];  %% composite slope using all positive and negative x's
  c2p=[]; c2n=[];  %% composite slope using 25% threshold for x's
  c3p=[]; c3n=[];  %% composite slope using 5% threshold for x's


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Iteration to generate the synthetic data for 1000 times
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

  nn=1000;
  for i=1:nn

      %-------------------------------------------------------
      % Generate sythetic 100 x-values with zero mean,
      % uniformly distributed in interval (-50,50).
      %-------------------------------------------------------
        x=-50 + (50-(-50))*rand(1,100);
        x=x-mean(x);

      %-------------------------------------------------------
      % Generate sythetic 100 noise values with the zero mean,
      % uniformly distributed in interval (-50,50).
      %-------------------------------------------------------
        e=-50 + (50-(-50))*rand(1,100);
        e=e-mean(e);

      %-------------------------------------------------------
      % Define asymetric y-time series with the asymmetry = 3:1
      %-------------------------------------------------------
        kpos=find(x>0);
        kneg=find(x<=0);

        y(kpos)=3*x(kpos)+e(kpos);
        y(kneg)=1*x(kneg)+e(kneg);
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  yp=y(kpos);
  yn=y(kneg);
  xp=x(kpos);
  xn=x(kneg);

%-------------------------------------------------------
% Calculate the anomaly Y time series
%-------------------------------------------------------
  y_mean=mean(y);
  YY=y-y_mean;

  YYp=yp-y_mean;
  YYn=yn-y_mean;



%-------------------------------------------------------
% Calculate symmetric regression
%-------------------------------------------------------
  p0(end+1)=(y*x')/(x*x');


%-------------------------------------------------------
% Asymetric regression model 1
% (i.e., removing means for entire x's andy's)
%-------------------------------------------------------
  p1p(end+1)=(YYp*xp')/(xp*xp');
  p1n(end+1)=(YYn*xn')/(xn*xn');


%-------------------------------------------------------
% Asymetric regression model 2:
% (i.e., removing means for xp's and Yp's,
%  and separately for xn's and Yn's)
%-------------------------------------------------------
  xn0=xn-mean(xn);
  xp0=xp-mean(xp);
  YYn0=YYn-mean(YYn);
  YYp0=YYp-mean(YYp);

  p2p(end+1)=(YYp0*xp0')/(xp0*xp0');
  p2n(end+1)=(YYn0*xn0')/(xn0*xn0');


%-------------------------------------------------------
% Composite for all positive x's and negative x's, respectively
%-------------------------------------------------------
  xnegcomp0=mean(xn);
  YYnegcomp0=mean(YYn);
  xposcomp0=mean(xp);
  YYposcomp0=mean(YYp);

  c1p(end+1)=YYposcomp0/xposcomp0;
  c1n(end+1)=YYnegcomp0/xnegcomp0;


%-------------------------------------------------------
% Composite for top and bottom 25% x's
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      %-----------------------------------------------------
        [xsort,ksort]=sort(x);
        xnegcomp=mean(x(ksort(1:25)));
        YYnegcomp=mean(YY(ksort(1:25)));
        xposcomp=mean(x(ksort(76:100)));
        YYposcomp=mean(YY(ksort(76:100)));

        c2p(end+1)=YYposcomp/xposcomp;
        c2n(end+1)=YYnegcomp/xnegcomp;


      %-----------------------------------------------------
      % Composite for top and bottom 5% x's
      %-----------------------------------------------------
        xnegcomp2=mean(x(ksort(1:5)));
        YYnegcomp2=mean(YY(ksort(1:5)));
        xposcomp2=mean(x(ksort(96:100)));
        YYposcomp2=mean(YY(ksort(96:100)));

        c3p(end+1)=YYposcomp2/xposcomp2;
        c3n(end+1)=YYnegcomp2/xnegcomp2;

  end


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Calculate the 90% uncertainty range for the slope estimates
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

   [psort,ksort]=sort(p0);
   P0bot5=psort(nn*0.05);
   P0top5=psort(nn*0.95);

   [psort,ksort]=sort(p1p);
   P1pbot5=psort(nn*0.05);
   P1ptop5=psort(nn*0.95);

   [psort,ksort]=sort(p1n);
   P1nbot5=psort(nn*0.05);
   P1ntop5=psort(nn*0.95);

   [psort,ksort]=sort(p2p);
   P2pbot5=psort(nn*0.05);
   P2ptop5=psort(nn*0.95);

   [psort,ksort]=sort(p2n);
   P2nbot5=psort(nn*0.05);
   P2ntop5=psort(nn*0.95);

   [psort,ksort]=sort(c1p);
   C1pbot5=psort(nn*0.05);
   C1ptop5=psort(nn*0.95);

   [psort,ksort]=sort(c1n);
   C1nbot5=psort(nn*0.05);
   C1ntop5=psort(nn*0.95);

   [psort,ksort]=sort(c2p);
```

```
    C2pbot5=psort(nn*0.05);
    C2ptop5=psort(nn*0.95);

    [psort,ksort]=sort(c2n);
    C2nbot5=psort(nn*0.05);
    C2ntop5=psort(nn*0.95);

    [psort,ksort]=sort(c3p);
    C3pbot5=psort(nn*0.05);
    C3ptop5=psort(nn*0.95);

    [psort,ksort]=sort(c3n);
    C3nbot5=psort(nn*0.05);
    C3ntop5=psort(nn*0.95);



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Plotting the scatter plot using the last iteration
%% and show the estimates of the seleccted slope estimates
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   figure;
   axes('position',[0.08 0.45 0.25 0.5]);

   h=plot(x,YY,'k.','color',[0.7 0.7 0.7],'MarkerSize',15);
   hold on;
   grid on;
   set(gca,'xlim',[-60 60],'xtick',[-60:30:60],'ylim',[-180 180],'ytick',[-180:60:180]);

   xpos=[1:55];
   xneg=-1*xpos;
   h1p=plot(xpos,xpos*mean(p1p),'r-','linewidth',1); hold on;
   h1n=plot(xneg,xneg*mean(p1n),'r-','linewidth',1); hold on;
   h2p=plot(xpos,(xpos-mean(xp))*mean(p2p)+mean(YYp),'k-','linewidth',1); hold on;
   h2n=plot(xneg,(xneg-mean(xn))*mean(p2n)+mean(YYn),'k-','linewidth',1); hold on;
   h3p=plot(xposcomp,xposcomp*mean(c2p),'ms','markersize',12,'markerfacecolor','m');
   h3n=plot(xnegcomp,xnegcomp*mean(c2n),'ms','markersize',12,'markerfacecolor','m');

   plot([0 0],[-180 180],'k-');
   plot([-60 60],[0 0],'k-');

   set(gca,'fontsize',10);
   xlabel('\bf\it x','FontName','times');
   ylabel('\bf\it Y','FontName','times');
   text(-55,160,'\bf (a)','FontSize',12);


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Plotting the estimates of the slopes
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

   axes('position',[0.38 0.45 0.3 0.5]);

   plot([0.5 6.5],[1 1],'k-','LineWidth',1,'Color',[0.7 0.7 0.7]); hold on;
   plot([0.5 6.5],[3 3],'k-','LineWidth',1,'Color',[0.7 0.7 0.7]);
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```
  h0=plot([1 1],[P0bot5 P0top5],'b-','linewidth',1); hold on; grid on;
  h0=plot(1,mean(p0),'b*'); hold on; grid on;

  h1p=plot([2 2],[P1pbot5 P1ptop5],'r-','linewidth',1); hold on; grid on;
  h1p=plot(2,mean(p1p),'rx');
  h1n=plot([2 2],[P1nbot5 P1ntop5],'r-','linewidth',1); hold on; grid on;
  h1n=plot(2,mean(p1n),'ro');

  h2p=plot([3 3],[P2pbot5 P2ptop5],'k-','linewidth',1); hold on; grid on;
  h2p=plot(3,mean(p2p),'kx');
  h2n=plot([3 3],[P2nbot5 P2ntop5],'k-','linewidth',1); hold on; grid on;
  h2n=plot(3,mean(p2n),'ko');

  h3p=plot([4 4],[C1pbot5 C1ptop5],'c-','linewidth',1); hold on; grid on;
  h3p=plot(4,mean(c1p),'cx');
  h3n=plot([4 4],[C1nbot5 C1ntop5],'c-','linewidth',1); hold on; grid on;
  h3n=plot(4,mean(c1n),'co');

  h4p=plot([5 5],[C2pbot5 C2ptop5],'m-','linewidth',1); hold on; grid on;
  h4p=plot(5,mean(c2p),'mx');
  h4n=plot([5 5],[C2nbot5 C2ntop5],'m-','linewidth',1); hold on; grid on;
  h4n=plot(5,mean(c2n),'mo');

  h5p=plot([6 6],[C3pbot5 C3ptop5],'g-','linewidth',1); hold on; grid on;
  h5p=plot(6,mean(c3p),'gx');
  h5n=plot([6 6],[C3nbot5 C3ntop5],'g-','linewidth',1); hold on; grid on;
  h5n=plot(6,mean(c3n),'go');

  ylabel('\bf Slope estimates');
  xlabel('\bf Method #''s');
  text(0.8,3.8,'\bf (b)','FontSize',12);

  set(gca,'xlim',[0.5 6.5],'xtick',[1:6]);
  set(gca,'xticklabel',[0 1 2 3 3 3]);
  set(gca,'ylim',[0 4]);


   hleg=legend([h0,h1p,h1n,h2p,h2n,h3p,h3n,h4p,h4n,h5p,h5n],'symmetric↙
regression','positive asymmetric regression 1','negative asymmetric regression↙
1','positive asymmetric regression 2', ...
            'negative asymmetric regression 2','all positive composite','all negative↙
composite', ...
            'top 25% composite','bottom 25% composite','top 5% composite','bottom 5%↙
composite','location','SouthOutside');
   set(gca,'position',[0.40 0.45 0.3 0.5]);
   set(hleg,'fontsize',8,'position',[0.71 0.45 0.28 0.3643]);
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